MSE440 Composite Technology 2024

Mechanics of composites

pierre-etienne.bourban@epfl.ch

Institut des matériaux (IMX)
Ecole Polytechnique Fédérale de Lausanne (EPFL),
CH-1015 Lausanne

Biblio

Les matériaux composites organiques
Traité des matériaux, vol 15, PPUR, Lausanne

Composites Materials, Design and applications
Daniel Gay, CRC Press 2014

Comprehensive composite materials,
Kelly A, Zweben C. ed., Elsevier 2000, 2017

Engineering mechanics of composites materials,
I.M. Daniel, O. Ishai, 2006

Design and manufacture of composite structures
G. Eckold, Woodhead publishing, 1994

Composite Materials, Step-by-step Projects,
Wanberg J.,Wolfgang Publications, 2014

Engineering with fibre-polymer laminates
P.C. Powell, Chapman & Hall, 1994

3-D textile reinforcements in composite materials,
A. Miravete editor, Woodhead publishing, 1999

CoDA ou LAP sur http://www.anaglyph.co.uk

http://www.efunda.com/formulae/solid_mechanics/composites/calc_ufrp_abd_layout.cfm

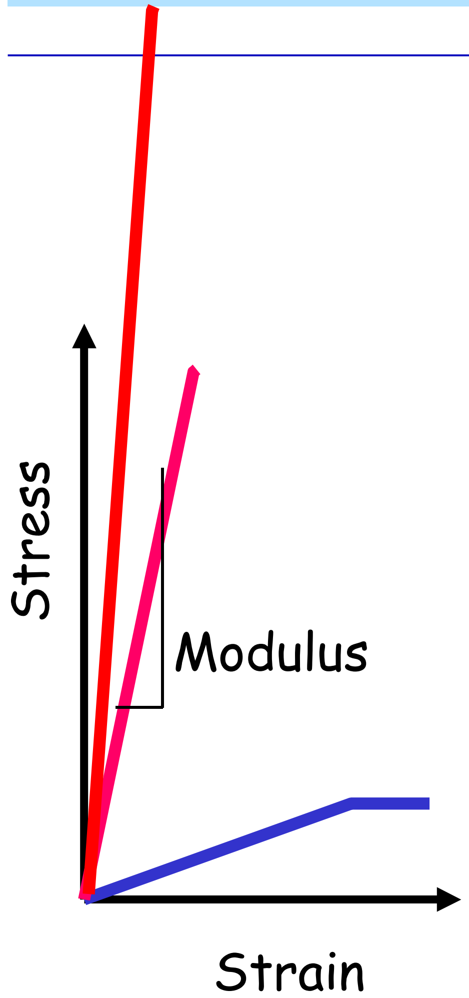
https://altairhyperworks.com/product/ESAComp

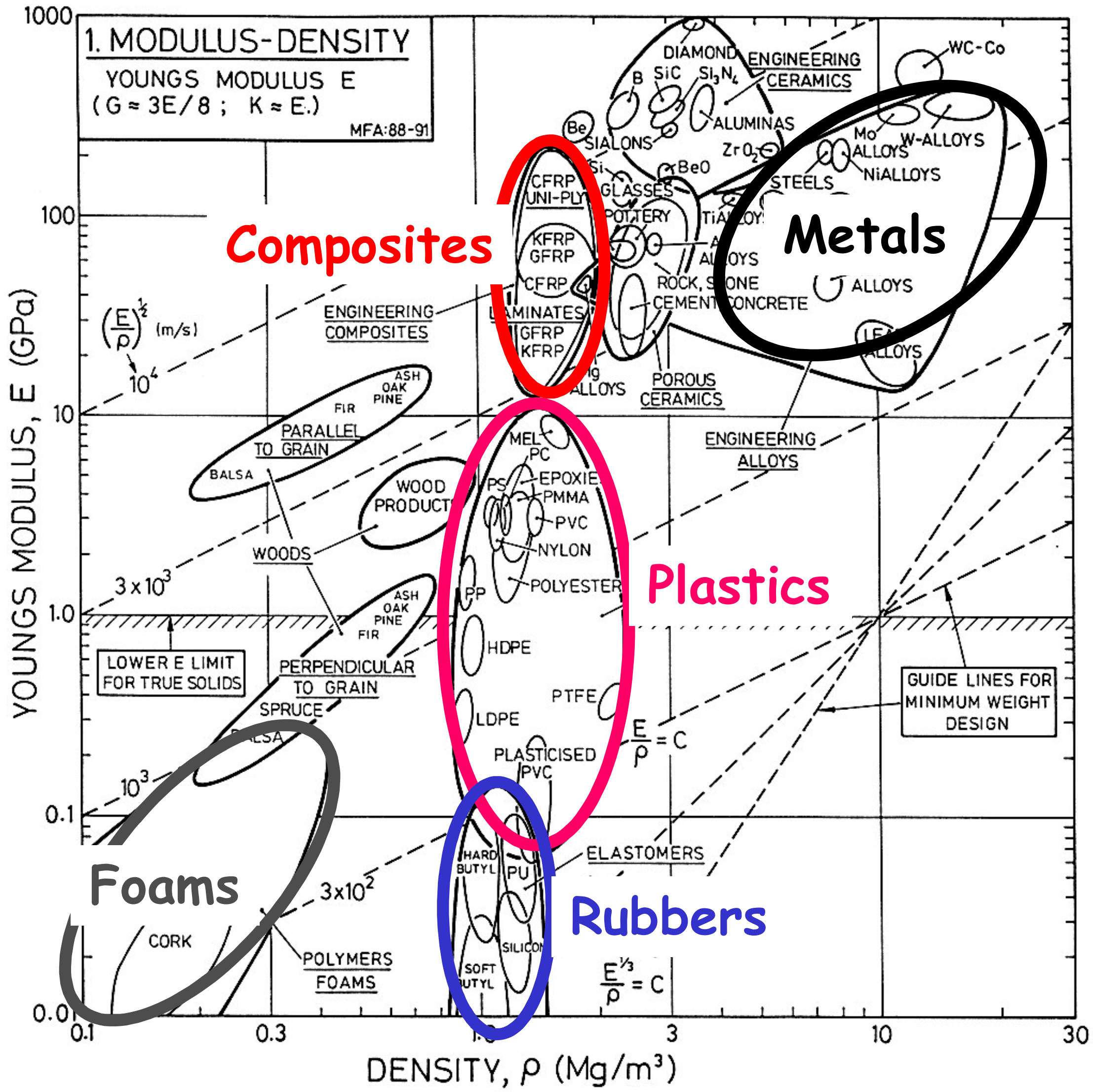
WiseTex sur http://wisetexdemo.software.informer.com/

LPAC Laboratory for Processing of Advanced Composites

EPFL

Siffness versus density





M.F. Ashby, Materials Selection for Mechanical design, Pergamon Press

LPAC
Laboratory for Processing of Advanced Composites

EPFL

Freedom in reinforcement types

Materials

Glass Aramid Steel Carbon

0 *E Modulus (Gpa)* *400*

Shapes

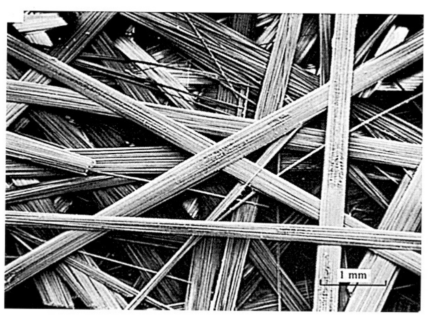
Particulates Fibres: short long discontinuous continuous

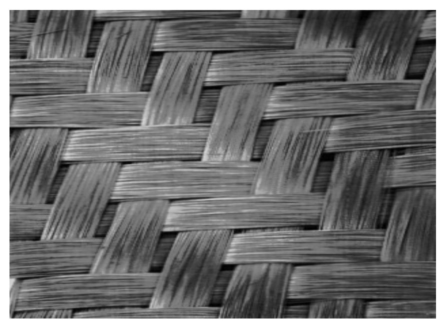
1 *Shape factors L/d* *∞*

Architectures

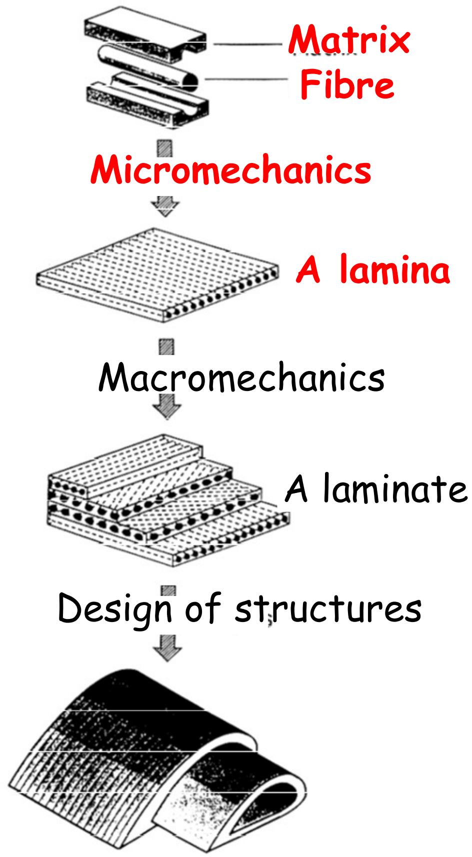
Random Knitts Weaves

0 *Orientation* *1*

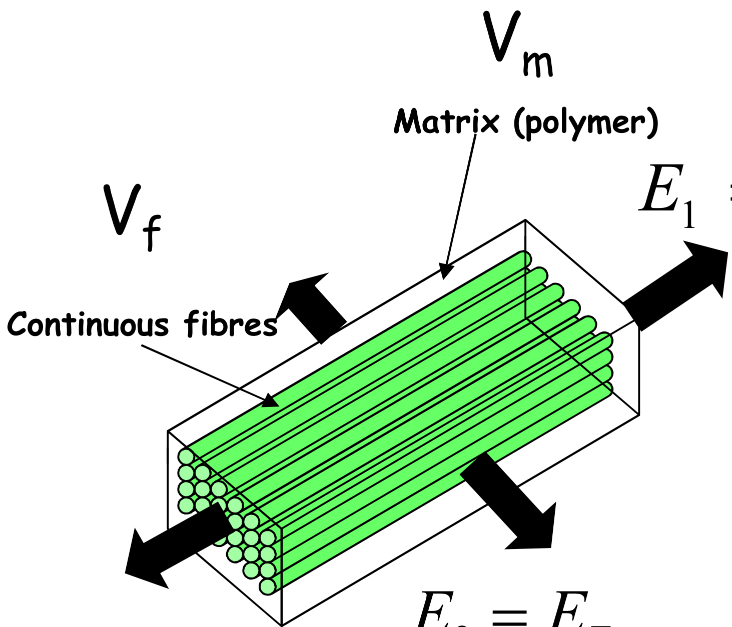




Micromechanics



Continuous fibres



$$E_1 = E_{Longitudinal} = \mathrm{E_f} V_f + \mathrm{E_m}(1 - V_f)$$

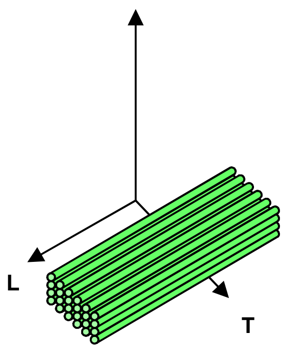
$$E_2 = E_{Transverse} = \frac{\mathrm{E_m} E_f}{E_f(1 - V_f) + \mathrm{E_m} V_f}$$

$$\nu_{LT} = V_f \nu_f + V_m \nu_m$$

$$\alpha_L = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}$$

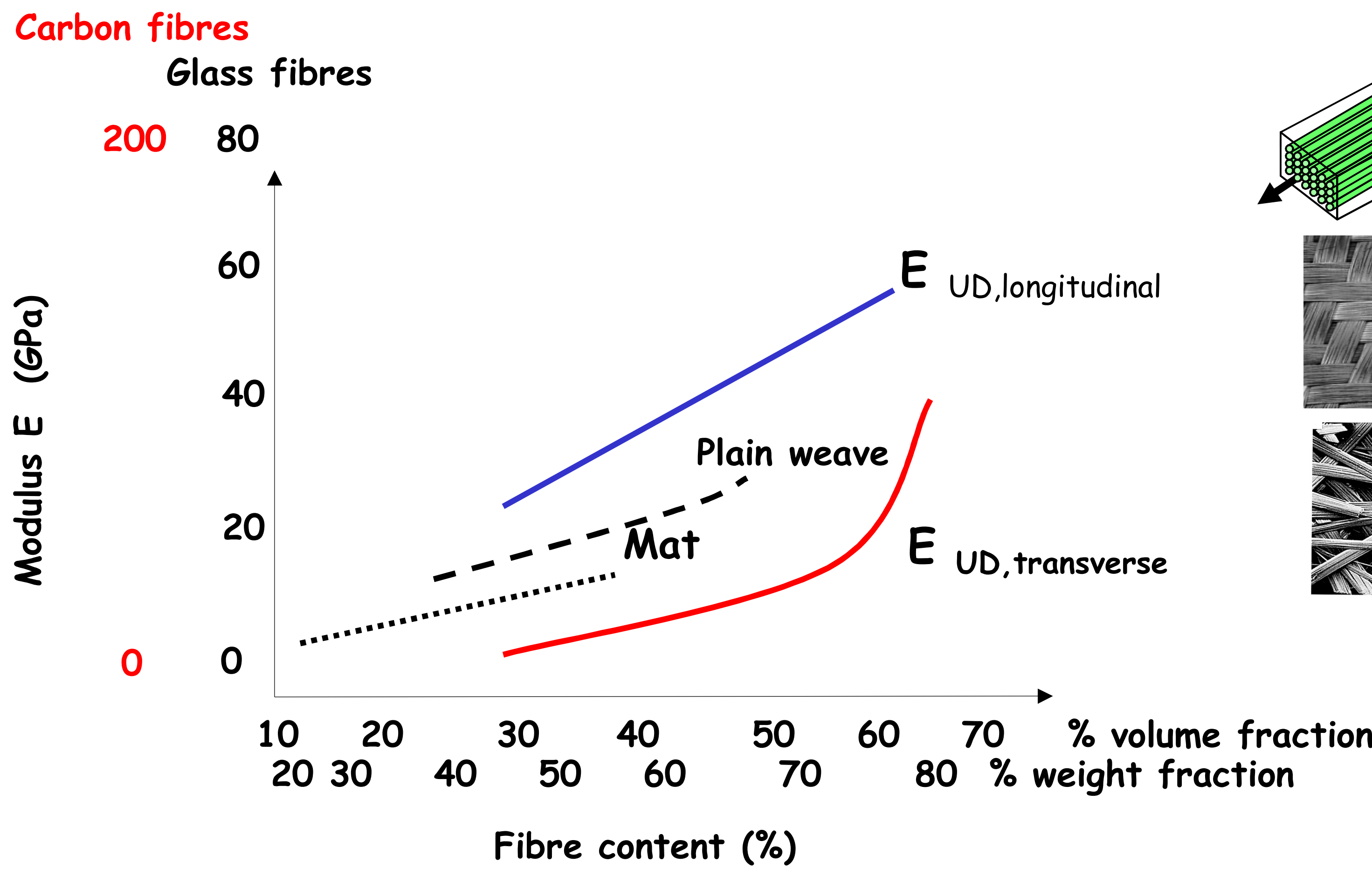
$$\alpha_T = (1 + \nu_m)\alpha_m V_m + (1 + \nu_f)\alpha_f V_f - \alpha_L \nu_{LT}$$

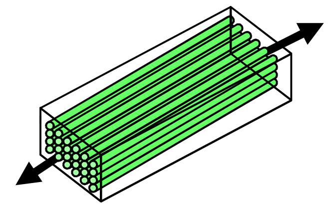
Properties of unidirectional composites

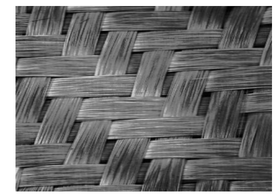


	S Glass Epoxy	Kevlar 49 Epoxy	Carbon HR Epoxy	Carbon HM Epoxy	Boron Epoxy
Vf	65 %	65 %	65 %	65 %	55 %
Density ρ	2.04 g/cm^3	1.36 g/cm^3	1.56 g/cm^3	1.5 g/cm^3	1.97 g/cm^3
E_L	56 GPa	86 GPa	145 GPa	270 GPa	220 GPa
E_T	16 GPa	5.6 GPa	10 GPa	7 GPa	2.3 GPa
ν_{LT}	0.26	0.32	0.29	0.3	0.26
G_{LT}	7 GPa	2.5 GPa	5.5 GPa	5.7 GPa	6.9 GPa
σ_{rL} **tension**	1.75 GPa	1.5 GPa	1.2 GPa	0.95 GPa	1.3 GPa
σ_{rT} **tension**	0.04 GPa	0.03 GPa	0.08 GPa	0.035 GPa	0.065 GPa
σ_{rL} **compression**	0.9 GPa	0.28 GPa	1 GPa	0.75 GPa	2.85 GPa
σ_{rT} **compression**	0.15 GPa	0.14 GPa	0.25 GPa	0.2 GPa	0.03 GPa
γ_{rLT} **shear**	0.06 GPa	0.05 GPa	0.1 GPa	0.055 GPa	0.06 GPa

Which stiffness do you need ?

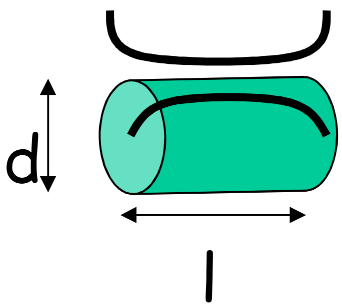






Short fibre composites

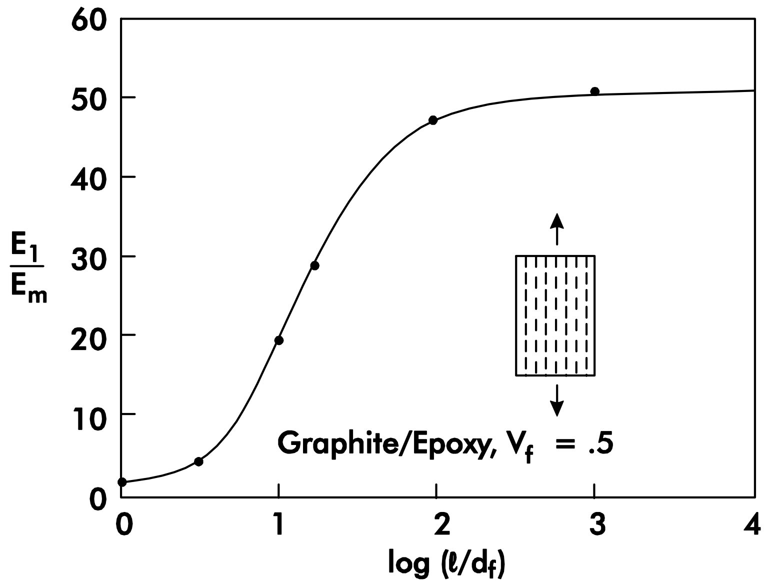
$$E = \eta_{\text{orientation}} \eta_{\text{fiber length}} E_f V_f + \mathrm{E_m}(1 - V_f)$$



$$l_c = \frac{d\,\sigma_{f,\,ult}}{2\,\tau_y}$$

Short fibre composites

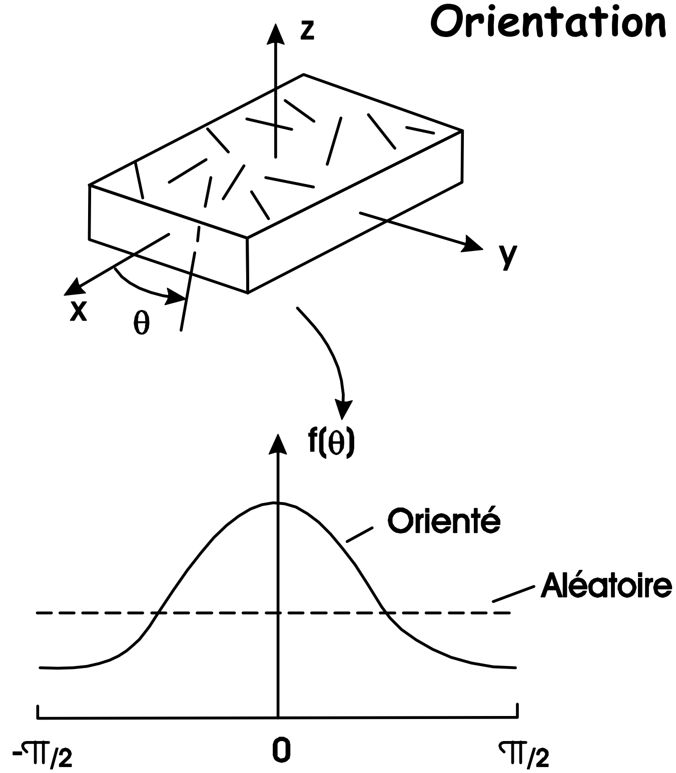
Length of fibres



$$E_I = E_f V_f \left[1 - \frac{\tanh(\beta\ell/2)}{\beta\ell/2}\right] + E_m V_m$$

Cox model

Orientation of fibres



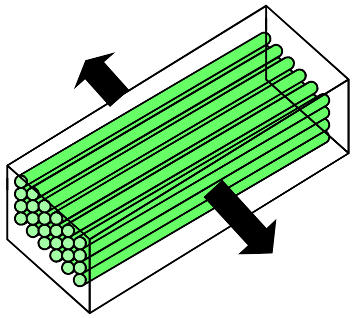
$$f(\theta) = \left(\frac{1}{\pi}\right) \frac{1 - \lambda^2}{1 + \lambda^2 - 2\lambda\cos(2\theta)}$$

where λ is the 'orientation parameter'. For random distribution, $\lambda=0$, for more oriented fibres , λ takes higher values, $0<\lambda<1$.

Laboratory for Processing of Advanced Composites

Halpin-Tsai equations

$$P = \frac{P_m(1+\xi\,\chi\,V_f)}{1-\chi\,V_f}$$



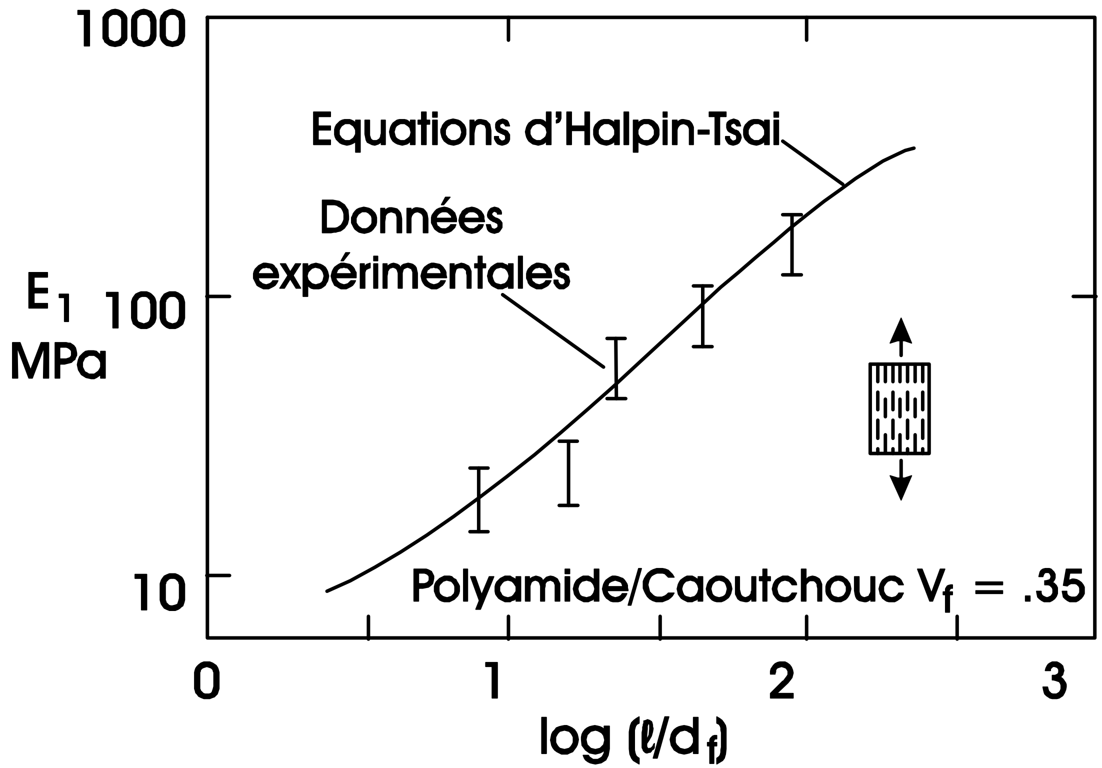
$$\xi(E_T) = 2$$

$$\xi(G_{LT}) = 1$$

$$\chi = \frac{P_f - P_m}{P_f + \xi\,\mathrm{P_m}}$$

$$\xi \approx \frac{2\mathrm{l}}{\mathrm{d}}$$

Short fibre composites



Properties of short fibre composites

Properties at 23°C	Zytel®		Zytel® 40% short fibres		Zytel® 50% short fibres	
	0 % HR	50% HR	0 % HR	50% HR	0 % HR	50% HR
Yiel stress σ_y (MPa)	84 MPa	48 MPa	205 MPa	135 MPa	230 MPa	155 MPa
Failure strain ε_y (%)	50 %	>300 %	3 %	6 %	2 %	5 %
Bending modulus E	2.7 GPa	0.9 GPa	10.5 GPa	6.5 GPa	23.5 GPa	8.5 GPa
Shock (notched) Izod	50 J/m	200 J/m	160 J/m	214 J/m	180 J/m	270 J/m
Shock resistance Charpy	Pas de rupture		60 kJ/m^2		65 kJ/m^2	
Density ρ	1.14 g/cm^3		1.45 g/cm^3		1.58 g/cm^3	
Melting temperature	245°C		233 °C		233 °C	
Heat distortion temperature in bending at 1.8 MPa	65 °C		224 °C			
Water absorption 24h (immersion)	1.6 %					
Molding shrinkage	1.3 %		0.18 %		0.16 %	

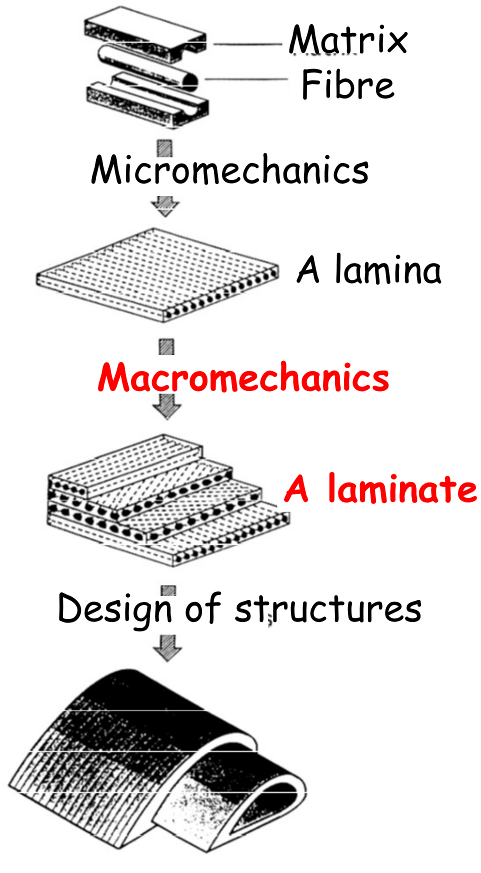
DUPONT The miracles of science™

matweb.com

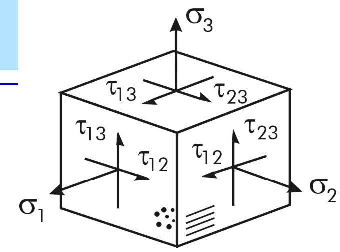
Other models

- with anisotropic fibres
- for 2D and 3D composites

Macromechanics



Introduction



$$\sigma_{ij} = f(\varepsilon_{kl})$$

Anisotropic materials — 81 csts

Linear elasticity

Hooke'slaw

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

Stresses and strains are symmetric — 36 csts

Tensor is symmetric — 21 csts

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

Material symmetry

- Monoclinical — 13 csts
- Orthotropy — 9 csts
- Transverse isotropy — 5 csts
- Isotropy — 2 csts

Approach used by the engineers

Constants of the engineer

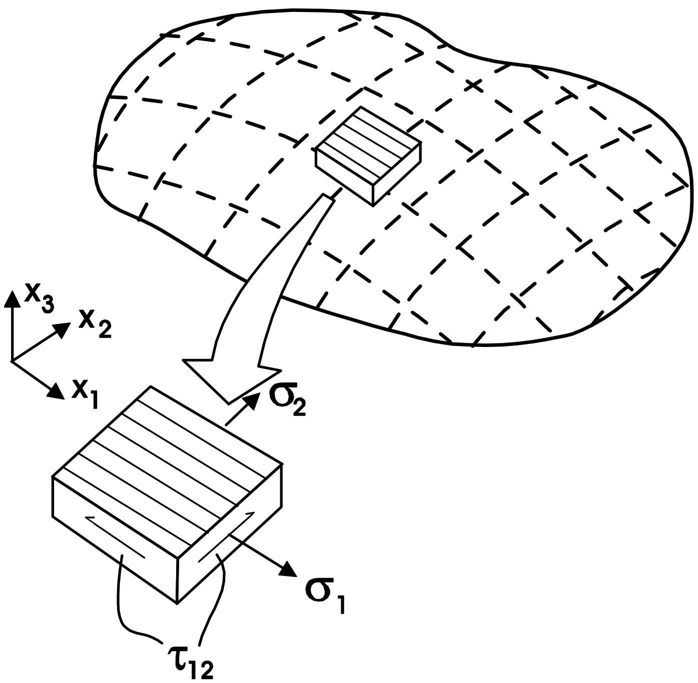
Orthotropic materials under plane stresses

Laminate theory and effective properties

Laminate symmetries and coupling effects

Design maps

Orthotropic materials under plane stresses



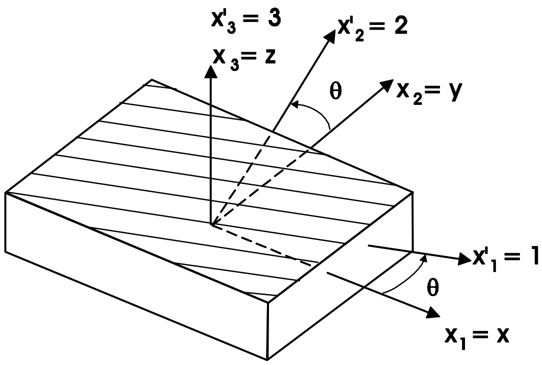
$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}$$

$$Q_{11}=\frac{E_1}{(1-\nu_{12}\nu_{21})} \qquad Q_{22}=\frac{E_2}{(1-\nu_{12}\nu_{21})}$$

$$Q_{12}=\frac{\nu_{12}E_2}{(1-\nu_{12}\nu_{21})}=\frac{\nu_{21}E_1}{(1-\nu_{12}\nu_{21})} \qquad Q_{66}=G_{12}$$

$$\left[S_{ij}\right]=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}=\begin{bmatrix}1/E_1 & -\upsilon_{21}/E_2 & 0\\ -\upsilon_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

Importance of fibre orientation



$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$$

m=cos, n=sin

$$\overline{Q}_{11}=m^4Q_{11}+2m^2n^2(Q_{12}+2Q_{66})+n^4Q_{22}$$

$$\overline{Q}_{21}=\overline{Q}_{12}=m^2n^2(Q_{11}+Q_{22}-4Q_{66})+Q_{12}(m^4+n^4)$$

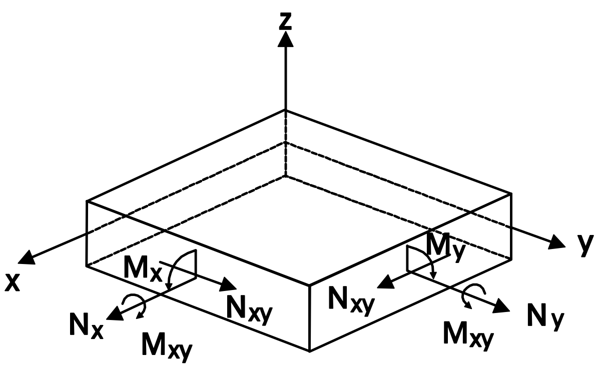
$$\overline{Q}_{22}=n^4Q_{11}+2m^2n^2(Q_{12}+2Q_{66})+m^4Q_{22}$$

$$\overline{Q}_{16}=m^3n(Q_{11}-Q_{12})+mn^3(Q_{12}-Q_{22})-2mn(m^2-n^2)Q_{66}$$

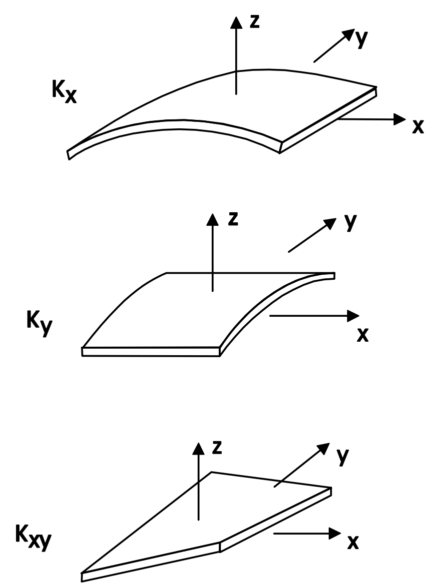
$$\overline{Q}_{26}=mn^3(Q_{11}-Q_{12})+m^3n(Q_{12}-Q_{22})+2mn(m^2-n^2)Q_{66}$$

$$\overline{Q}_{66}=m^2n^2(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})+(m^4+n^4)Q_{66}$$

Laminate elasticity



$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}$$

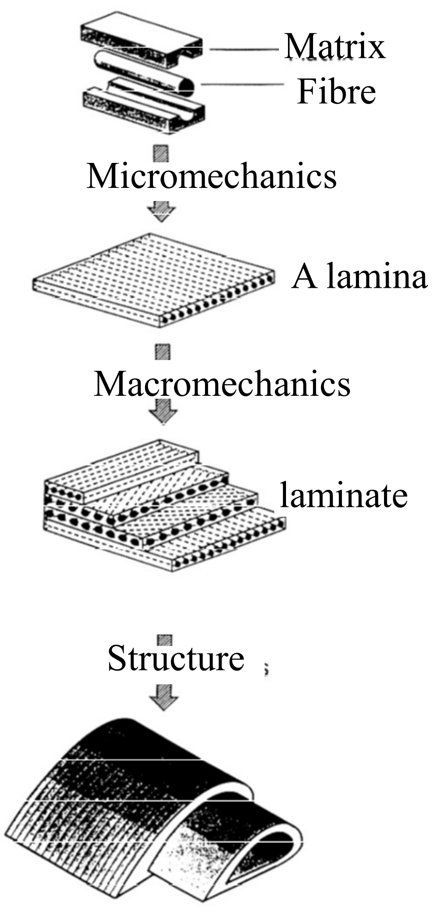


$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k - z_{k-1}\right) \qquad B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k^2 - z_{k-1}^2\right) \qquad D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

From fibre to structures



$$E_1 = E_f V_f + E_m (1 - V_f) \qquad P = \frac{P_m (1 + \xi \chi V_f)}{1 - \chi V_f}$$

$$Q_{11} = \frac{E_1}{(1 - \nu_{12}\nu_{21})}$$

$$\bar{Q}_{11} = m^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \qquad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k - z_{k-1})$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

$$E_x = \frac{A_{11} A_{22} - A_{12}^2}{h A_{22}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Elastic materials
No porosity
Interfaces with good adhesion of matrix to fibres

Fibes orientations

Orthotropy
Plane stress

Good adhesion bewteen plies

Symmetry in stacking sequence

Coupling effects

Load case
Effective properties
Mechanics of materials

Symmetric and balanced laminates

Laminate	A_{11} (MN/m)	A_{22} (MN/m)	G_{xy} (GPa)
$[90_8]_s$	11.09	153.28	2.29
$[0/90_7]_s$	28.86	135.51	2.29
$[0_2/90_6]_s$	46.64	117.73	2.29
$[0_3/90_5]_s$	64.41	99.96	2.29
$[0_4/90_4]_s$	82.18	82.18	2.29
$[0_5/90_3]_s$	99.96	64.41	2.29
$[0_6/90_2]_s$	117.73	46.64	2.29
$[0_7/90]_s$	135.51	28.86	2.29
$[0_8]_s$	153.28	11.09	2.29
$[\pm45/90_6]_s$	20.21	126.85	6.62
$[\pm45/0/90_5]_s$	37.98	109.08	6.62
$[\pm45/0_2/90_4]_s$	55.76	91.31	6.62
$[\pm45/0_3/90_3]_s$	73.53	73.53	6.62
$[\pm45/0_4/90_2]_s$	91.31	55.76	6.62
$[\pm45/0_5/90]_s$	109.08	37.98	6.62
$[\pm45/0_6]_s$	126.85	20.21	6.62
$[\pm45_2/90_4]_s$	29.33	100.43	10.95
$[\pm45_2/0/90_3]_s$	47.11	82.65	10.95
$[\pm45_2/0_2/90_2]_s$	64.88	64.88	10.95
$[\pm45_2/0_3/90]_s$	82.65	47.11	10.95
$[\pm45_2/0_4]_s$	100.43	29.33	10.95
$[\pm45_3/90_2]_s$	38.45	74.00	15.27 F
$[\pm45_3/0/90]_s$	56.23	56.23	15.27 F
$[\pm45_3/0_2]_s$	74.00	38.45	15.28 F
$[\pm45_4]_s$	47.58	47.58	19.60 F

Effective properties

Symmetric and balanced

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

Only mechanical loading
Uniform strain through the thickness

$$\varepsilon_x^0 = a_{11} N_x = \varepsilon_x$$

$$\sigma_x = \frac{N_x}{h}$$

$$\varepsilon_x = a_{11}\, h\, \sigma_x \qquad E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{h\, a_{11}} \qquad E_x = \frac{A_{11}A_{22} - A_{12}^2}{h\, A_{22}}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Effective engineering properties

to be used in material mechanics equations

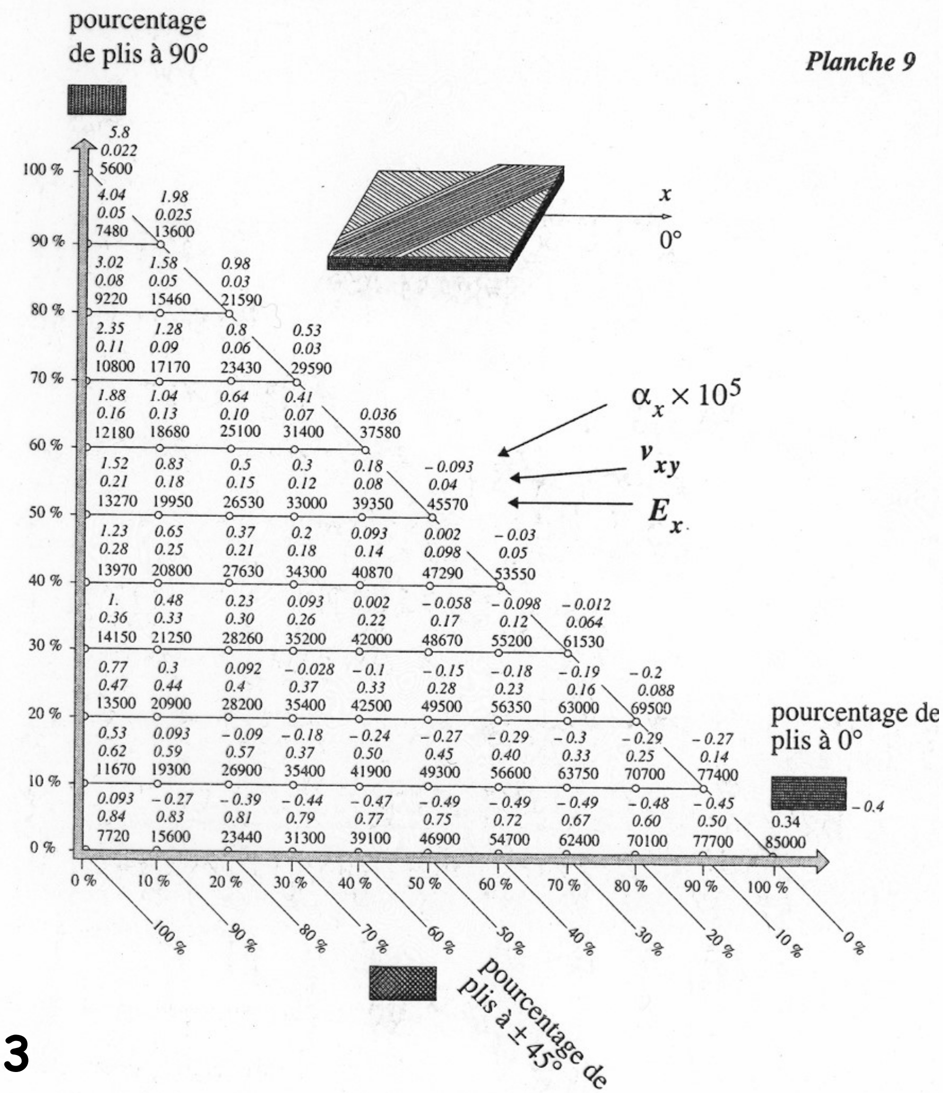
Stratifié kevlar/époxyde

V_f = 60 % de fibres en volume

épaisseur du pli : 0.13 mm

caractéristiques du pli : *cf.* paragraphe 3.3.3.

Planche 9



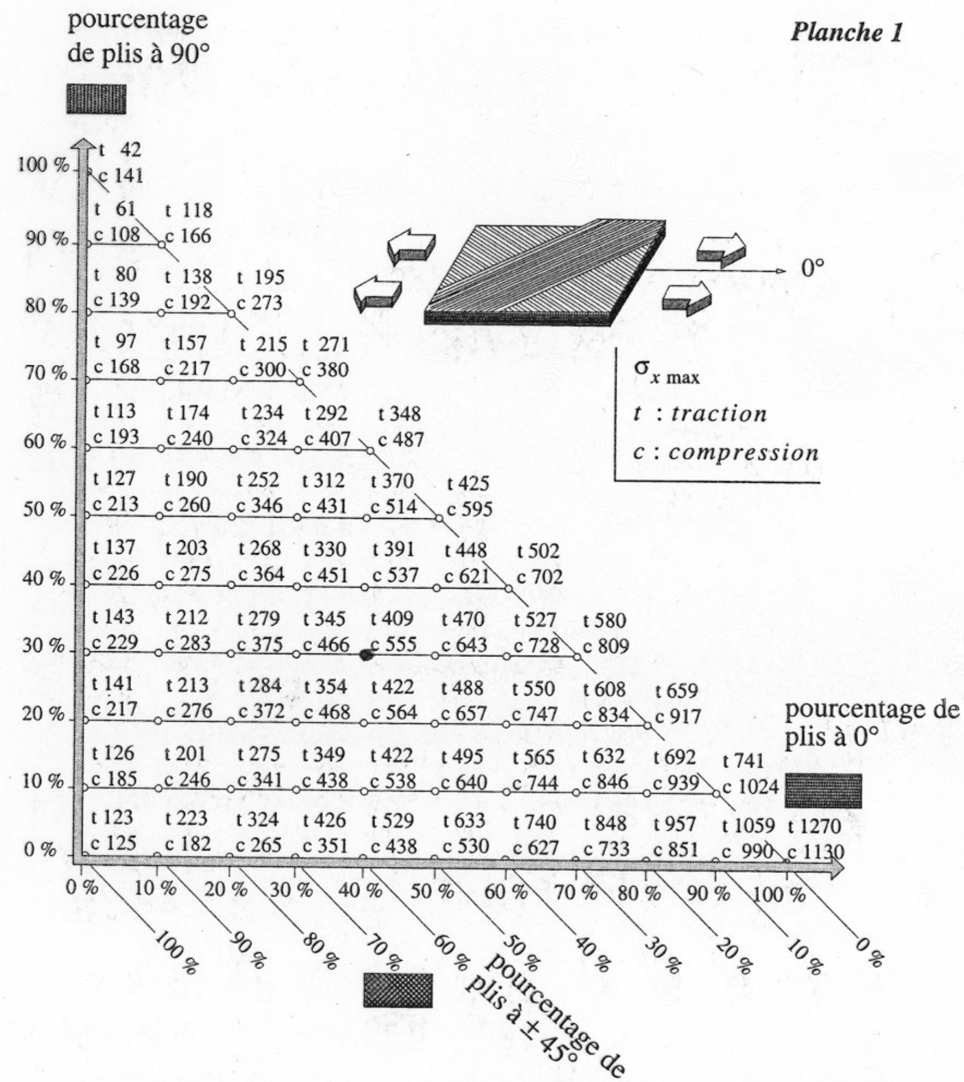
Stratifié carbone/époxyde

V_f = 60 % de fibres en volume

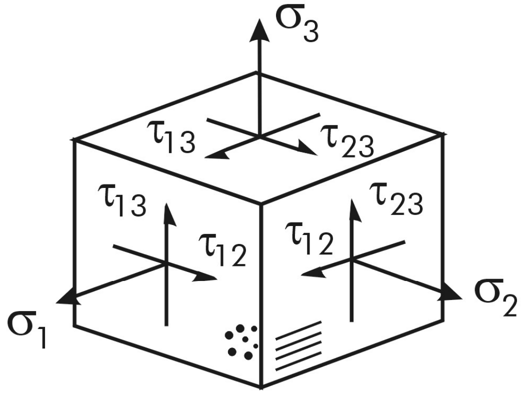
épaisseur du pli : 0.13 mm

caractéristiques du pli : modules, contraintes de rupture : *cf.* paragraphe 3.3.3.

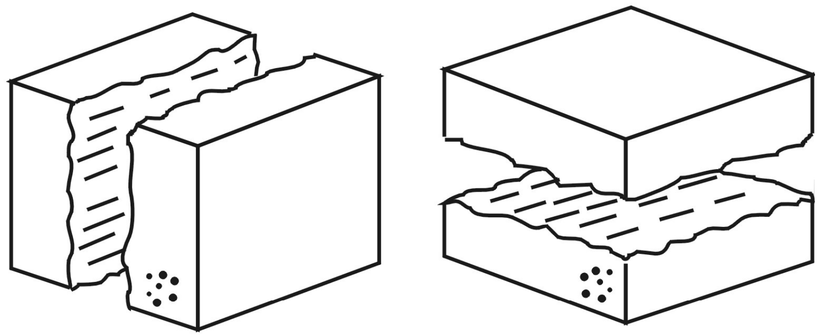
Planche 1



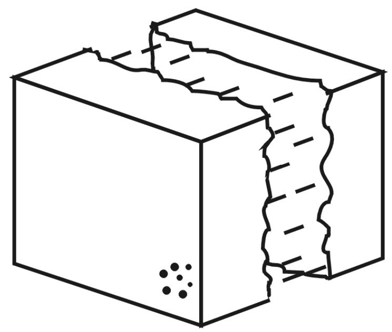
Failure of composites



(a) state of stresses

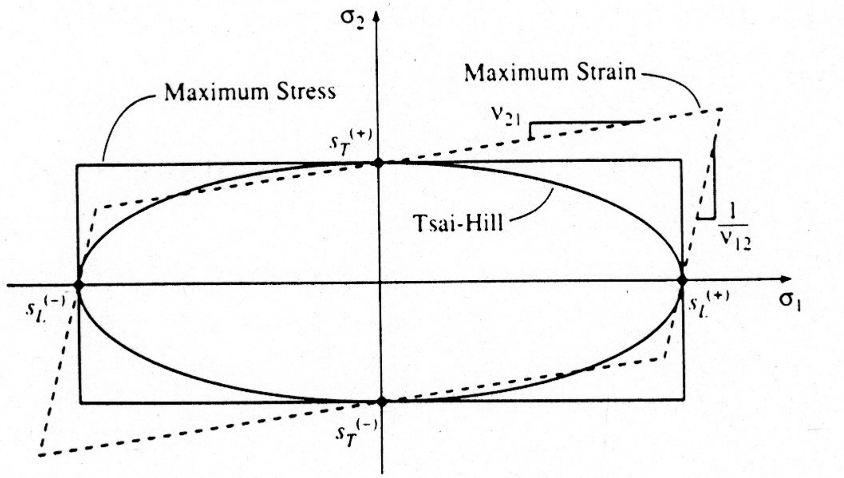


(b) matrix rupture

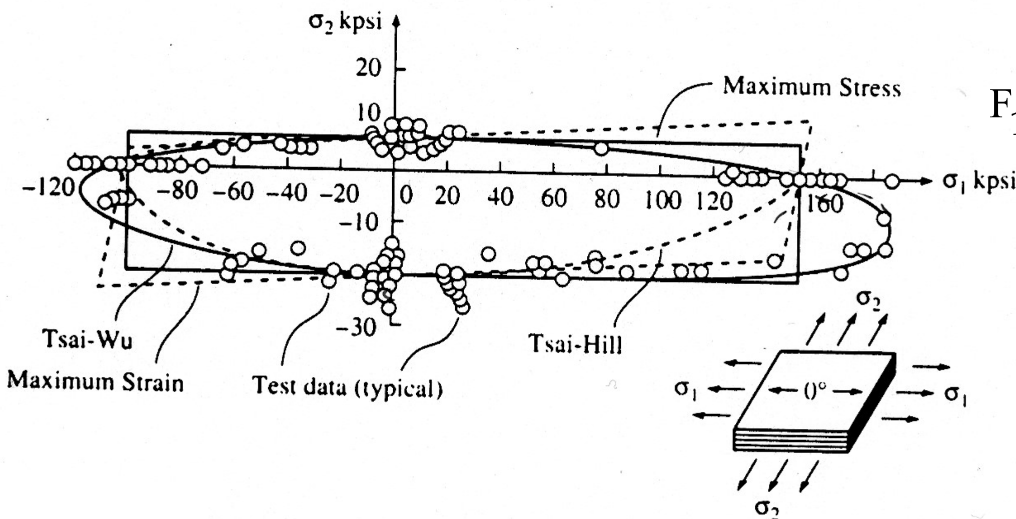


(c) decohesion, rupture of the fibes

Failure criteria



$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\left(X_1^T\right)^2} + \frac{\sigma_2^2}{\left(X_2^T\right)^2} + \frac{\tau_{12}^2}{S_6^2} = 1$$

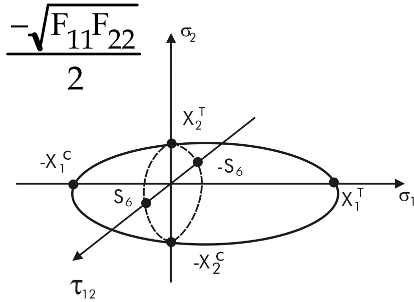


$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_1\sigma_2 = 1$$

$$F_1 = \frac{1}{X_1^T} - \frac{1}{X_1^C} \qquad F_{11} = \frac{1}{X_1^T X_1^C} \qquad F_{12} = \frac{-\sqrt{F_{11}F_{22}}}{2}$$

$$F_2 = \frac{1}{X_2^T} - \frac{1}{X_2^C} \qquad F_{22} = \frac{1}{X_2^T X_2^C}$$

$$F_{66} = \frac{1}{S_6^2}$$



First ply and final failures

Stresses in the plies

$$[\sigma] = [\overline{Q}]\left(\{\varepsilon^0\} + z\ \{\kappa\}\right)$$

1

2,3…

New stress conditions

Failure criteria

$$\frac{\sigma_1^2 - \sigma_1\sigma_2}{\left(\sigma_{u,1}^T\right)^2} + \frac{\sigma_2^2}{\left(\sigma_{u,2}^T\right)^2} + \frac{\tau_{12}^2}{\tau_u^2} = 1$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

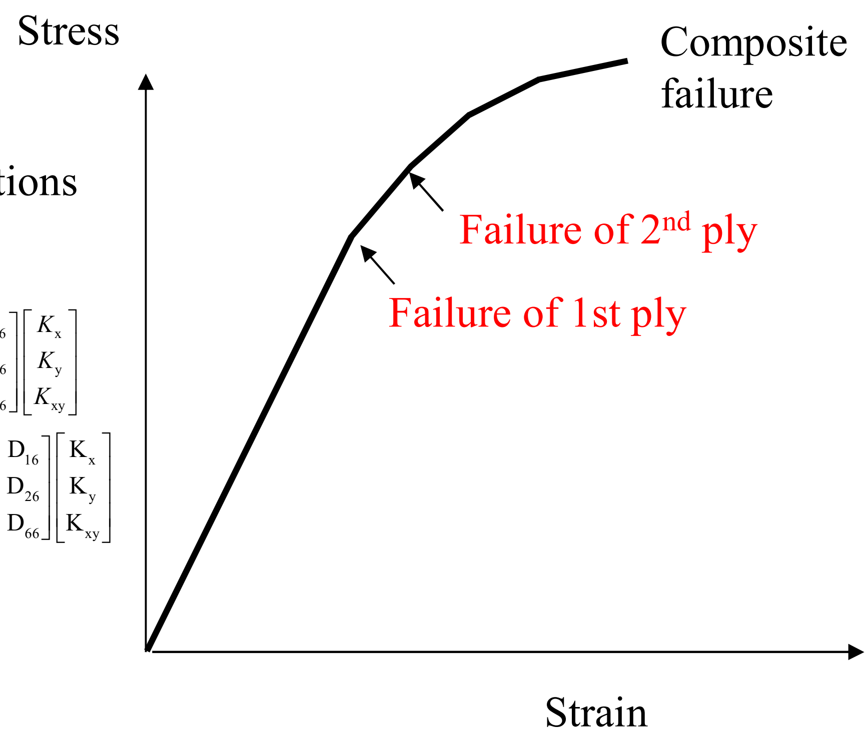
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

Failure of 1 ply

$$\left(\overline{Q_{ij}}\right)_{\text{pli rompu}} = 0$$

$$\boxed{\begin{aligned} A_{ij} &= \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k (z_k^3 - z_{k-1}^3) \end{aligned}}$$

Re-calculation

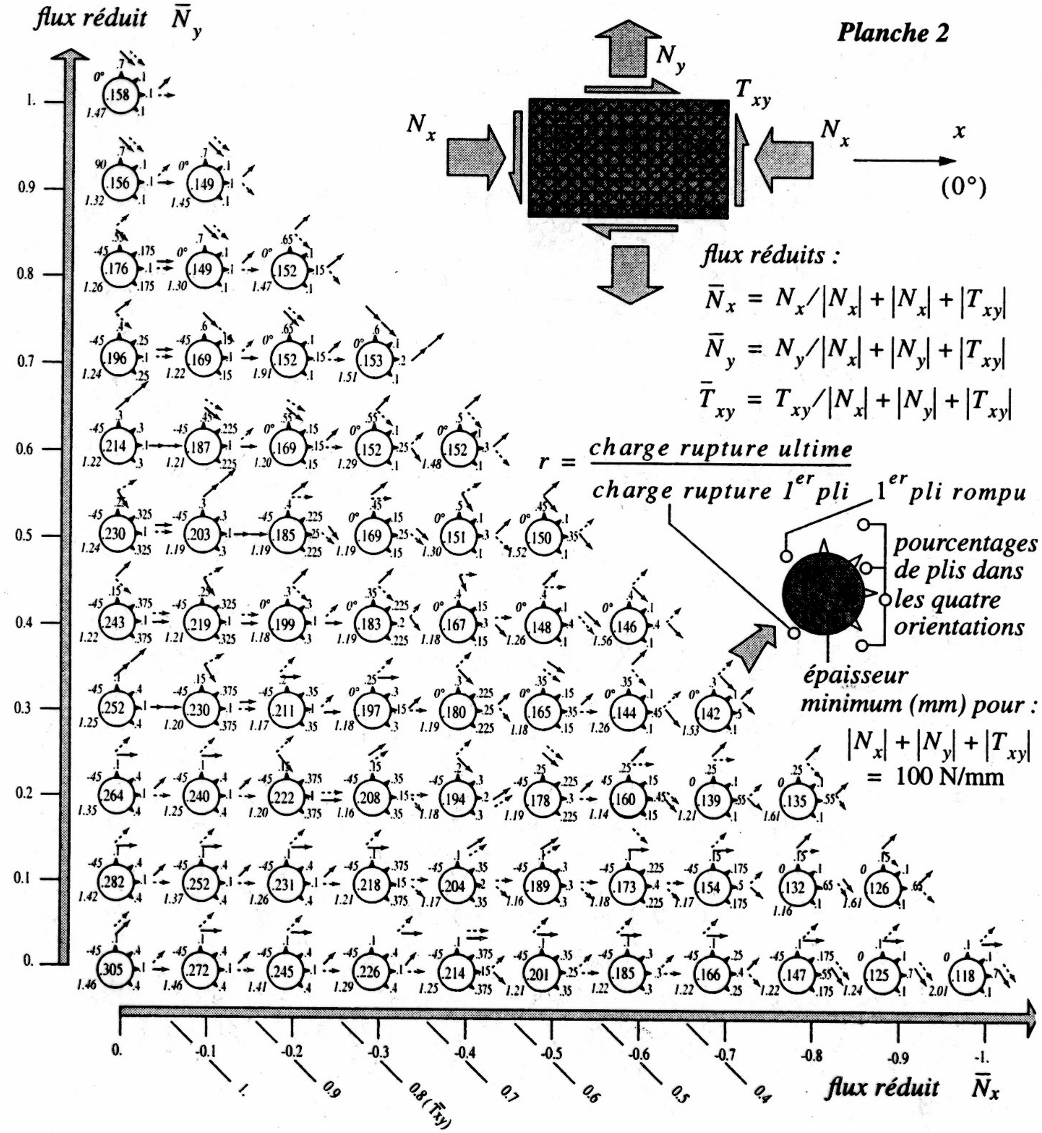


Design maps

Composition optimum d'un stratifié carbone/époxyde

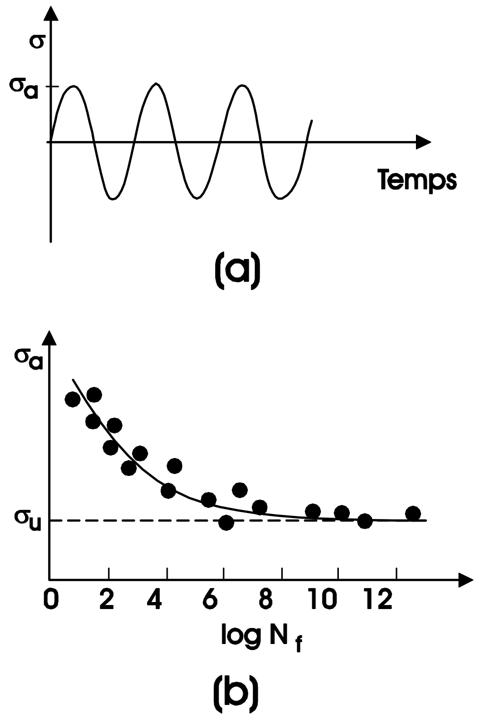
$V_f = 0{,}6$; caractéristiques du pli : *cf.* annexe 1 ou paragraphe 3.3.3.

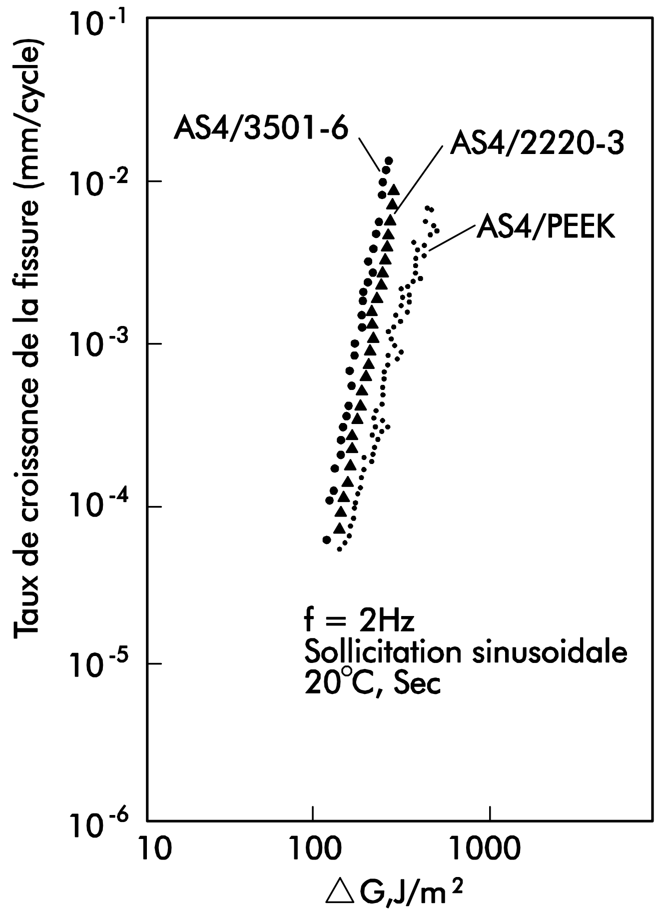
10 % minimum de plis dans chaque direction 0°, 90°, + 45°, – 45°



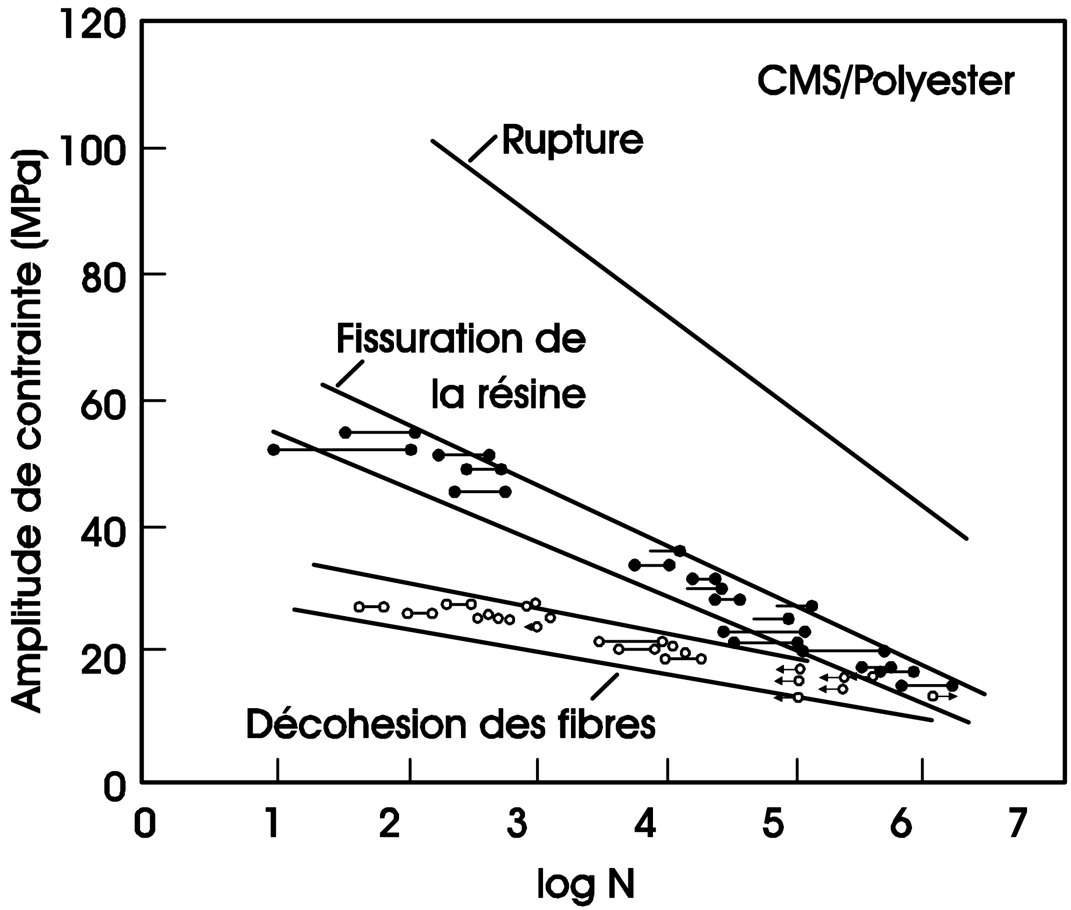
Laboratory for Processing of Advanced Composites

Fatigue of composites



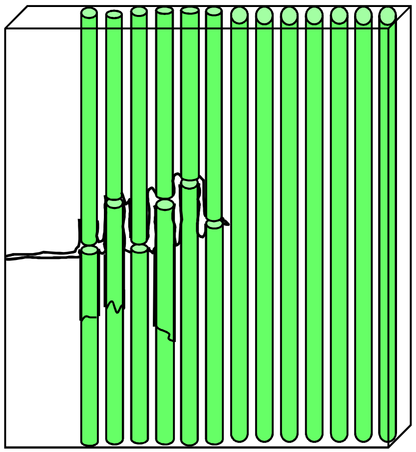


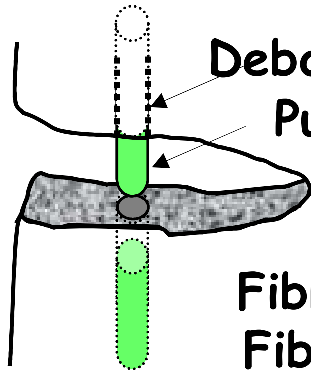
Fatigue of composites



Damage in composites

Matrix rupture



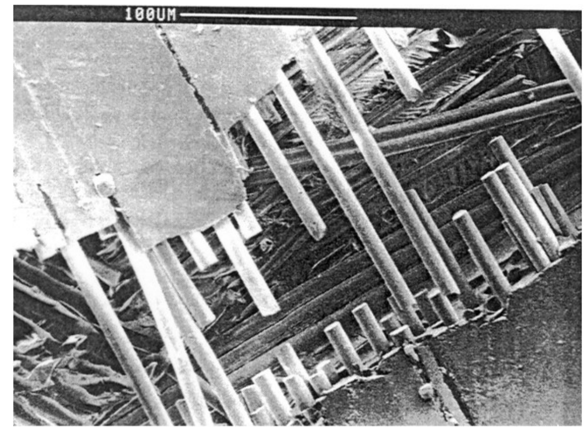


Debonding

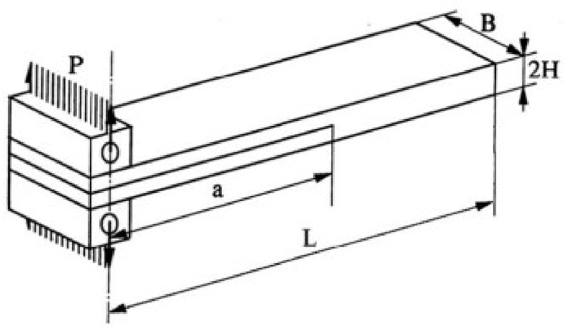
Pull-out

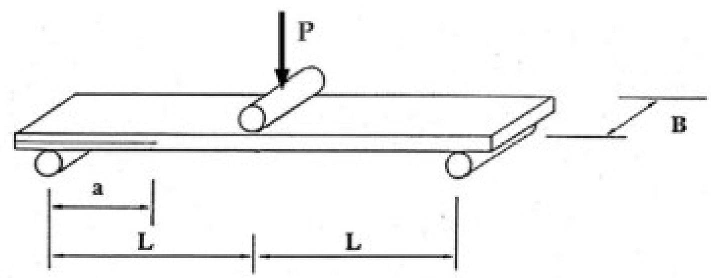
Fibre bridging

Fibre rupture

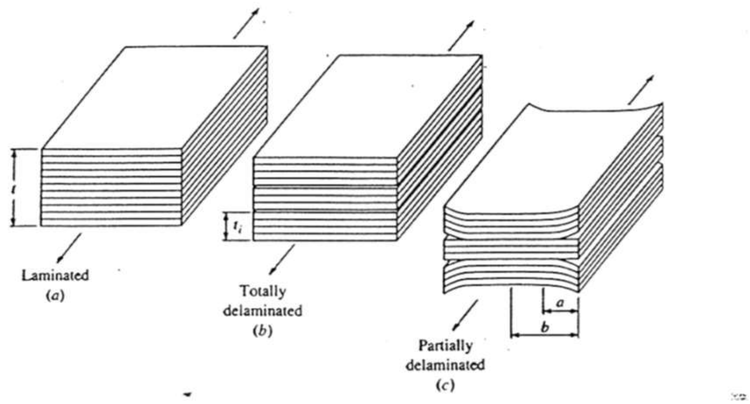


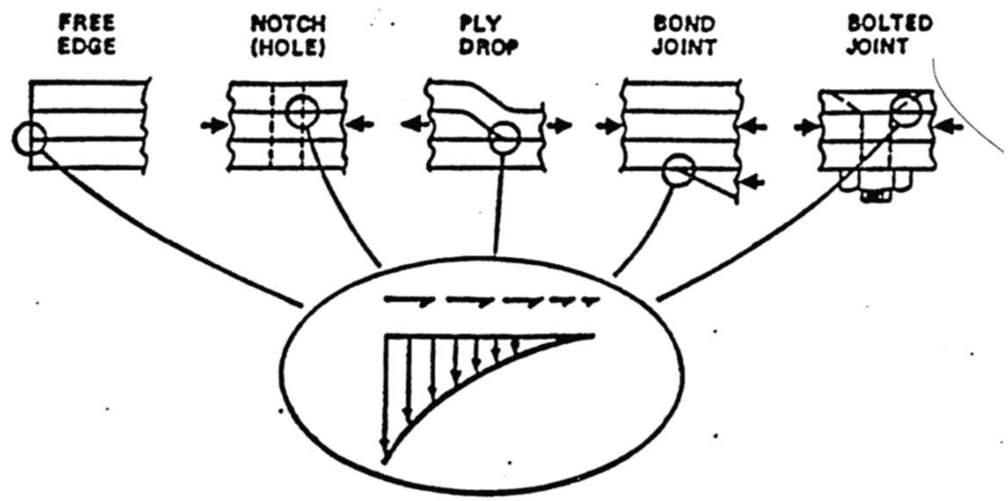
Energy release rate G, modes I, II crack propagations

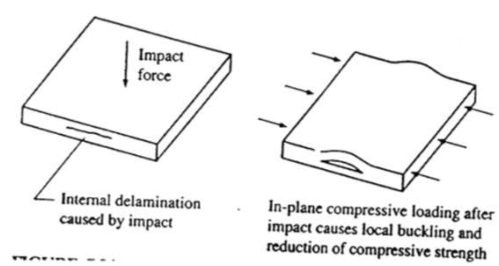




Delamination







Laboratory for Processing of Advanced Composites

Design

Micro and macromechanics

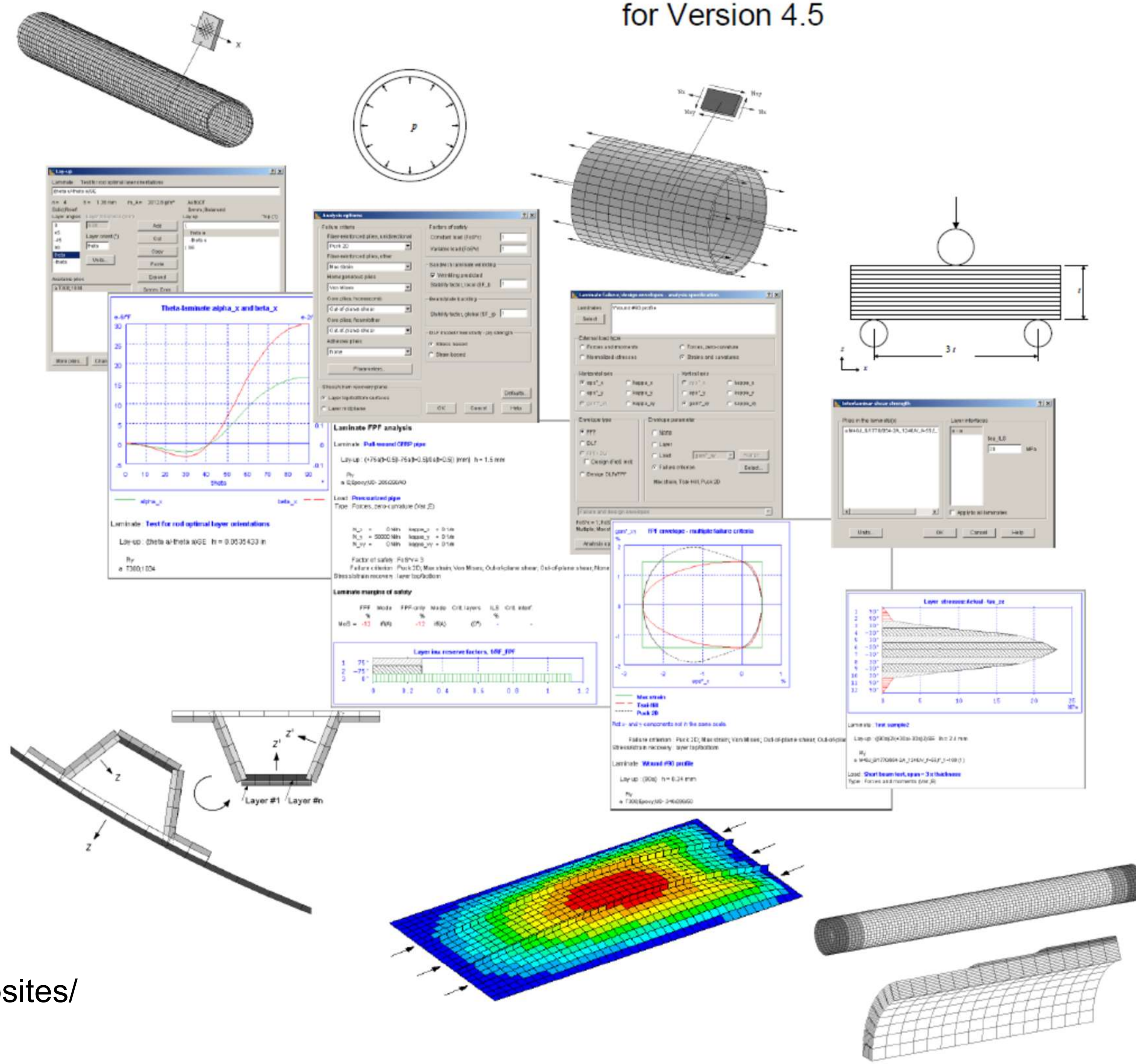
Mechanical tests

CADFEM tools

Applications

Layered composites

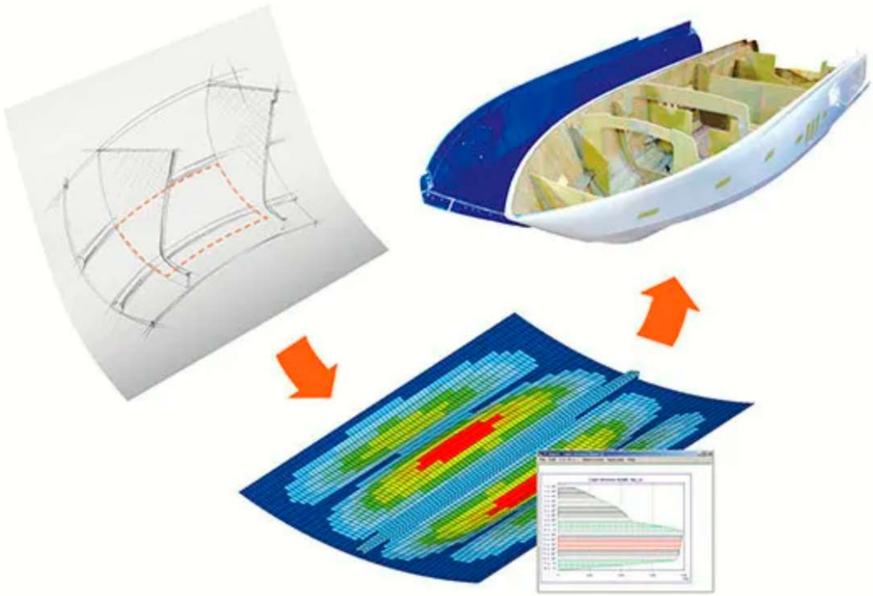
for Version 4.5

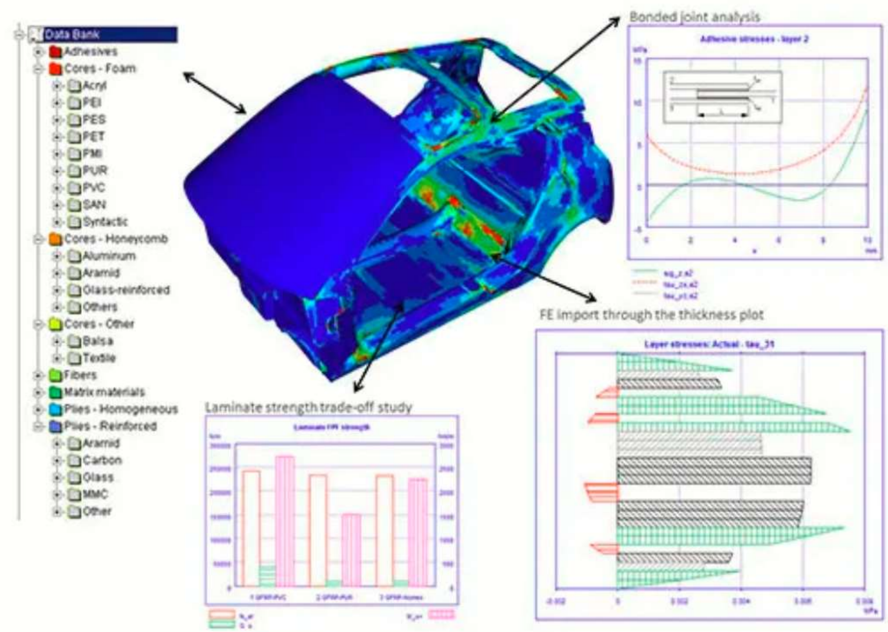


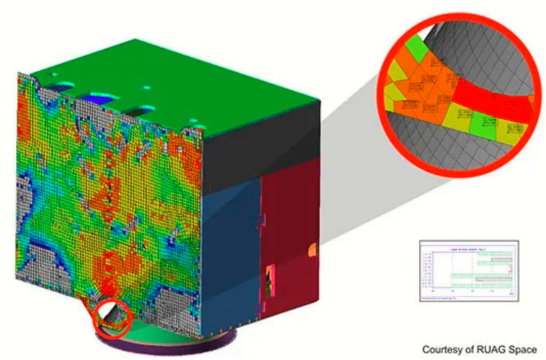
ESAComp Example Cases

https://www.altair.com/composites/

FEM







https://filecr.com/windows/esacomp/?id=13436849765

Multiscale analysis

Example: Prof. Sung Kyu Ha, Hanyang University

Micro-meso-macro scale
Impact simulation
Damage in thin and thick laminates
Fatigue life prediction
Blade design

More to come

Textile composites

Exercises on Moodle